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JEL classification: C51; G12; G23

Keywords: Private equity; Liquidity risk; Cost of capital

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Private Equity Performance and Liquidity Risk

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Abstract

This is the first study that provides evidence of liquidity risk in a large sample of private equity investments. It relies on the realized cash flows of 4,403 liquidated investments. We find that a one standard deviation increase in unexpected aggregate liquidity raises returns between 4% and 10% annually, depending on liquidity measures. This effect is robust to controlling for investment characteristics and macroeconomic variables. Larger investments and investments from more mature private equity firms have returns that are more sensitive to unexpected liquidity. Using the Pástor and Stambaugh (2003) traded liquidity factor, we estimate a liquidity risk premium in private equity of about 3% annually. Accounting for liquidity risk, the historical cost of capital for private equity is about 24% annually and the alpha (before fees) is close to zero.

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1 Introduction

Recent literature finds that liquidity risk is priced in public equity (e.g., Pástor and Stambaugh (2003), Acharya and Pedersen (2005), Sadka (2006), and Bekaert, Harvey, and Lundblad (2007)). This means that stocks whose returns are more sensitive to aggregate liquidity have higher average returns. Pástor and Stambaugh (2003, p.683) argue that "[...] it would also be useful to explore whether some form of systematic liquidity risk is priced in other financial markets". This paper investigates whether liquidity risk is a significant determinant of returns in private equity.[1]

It is important to emphasize from the start that the focus of this paper is on the compensation for systematic risk originating from time-varying liquidity, and not on the asset-specific liquidity characteristic (the liquidity level).

Liquidity risk is likely to be of first order importance in private equity for at least three reasons. First, Acharya and Pedersen (2005) argue that liquidity risk originates from uncertainty in transaction costs. As private equity funds often sell entire corporations, they may face larger uncertainty in transaction costs than public equity investors making a trade of similar size.

Second, private equity investors may have a higher tolerance for liquidity risk. Catering to these investors, a private equity fund may tilt its portfolio towards companies with larger exposure to this source of risk. As a result, private equity returns are likely to load more heavily on liquidity risk.

Third, private equity investments are highly levered and often need to be refinanced. Providers of debt to private equity – mainly banks and hedge funds – are sensitive to changes in funding liquidity (Brunnermeier and Pedersen (2009)). When aggregate liquidity is low, creditors may choose to force private equity investments into bankruptcy rather than providing new finance. Hence, returns may be lower in times of low liquidity (all else equal).

Anecdotal evidence from the recent financial crisis is consistent with the view that

[1] Because of data availability, the definition of private equity, here, is restricted to leveraged buyout investments (including management buy-out and buy-in) and growth investments made by private equity funds. According to Kaplan and Stromberg (2009) the asset value of these investments (past and present) totals $1.6 trillion.

private equity returns are particularly sensitive to liquidity. Dividends from private equity funds went from record highs in 2006 and the first half of 2007 (when liquidity was at record highs) to virtually zero for the second half of 2007 and 2008 (when liquidity was at record lows). This simple evidence does not mean that returns have followed the same pattern, but it underscores the importance of quantifying liquidity risk in private equity.

This study is the first to provide evidence of liquidity risk in a large sample of private equity investments. Importantly, we rely on realized cash flows– and not on subjective net asset values (NAVs) – of 4,403 liquidated investments. Having exact cash flows and a large number of liquidated investments allows us to obtain robust performance measures and risk estimates. In addition, our dataset presents only a minor sample selection bias since it contains both successful and unsuccessful investments (about 10% of the investments in our dataset are bankrupt).

We begin by documenting the importance of market-wide liquidity shocks for private equity returns. Returns are measured using a Modified IRR, which assumes that intermediate cash flows are reinvested at the S&P 500 rate of return. For liquidity shocks, we use three prominent measures: those of Pástor and Stambaugh (2003), Acharya and Pedersen (2005) and Sadka (2006). Next, we compute the average liquidity shock during the life of each investment. This measures whether the market-wide liquidity was surprisingly high or low between the starting date of the investment and the exit date of the investment. We label this variable the 'liquidity conditions' faced by an investment.

We find that a one-standard deviation improvement in liquidity conditions increases returns by about 10% annually when using the Pastor-Stambaugh measure, about 6% when using the Acharya-Perdersen measure, and about 4% when using the Sadka measure. Irrespective of the liquidity measure that we use, this effect is statistically significant.

We also investigate the interaction of liquidity with other variables. The argument for liquidity risk based on transaction costs (Acharya and Pedersen (2005)) suggests that larger investments are more sensitive to this source of risk. We find it to be the case, especially when using the Acharya-Pedersen and Sadka measures. The evidence also suggests that the returns of more mature private equity firms are more sensitive to liquidity. This may be the result of older firms being more levered (James and Demiroglu (2009)), or older firms having investors with deeper pockets, hence being more risk tolerant, or both.

Liquidity conditions are certainly related to a number of macroeconomic variables. Chen, Roll, and Ross (1986) show that default spreads and growth in industrial production are related to asset returns. We thus control for these variables and find that growth in industrial production is consistently and positively related to returns, but liquidity remains significant. Next, we control for volatility shocks in order to address the argument made by Spiegel and Wang (2005) and Bandi, Moise, and Russel (2008) that aggregate liquidity and aggregate volatility are closely related. We find that volatility does not affect the relation between liquidity and performance. Similarly, Cumming, Flemming, and Schwienbacher (2005) argue that IPO volume and liquidity are related, but controlling for the innovation in number of IPOs leaves the liquidity effect intact too.

The above results show that private equity investments offer superior returns when they are held during times of positive shocks to aggregate liquidity. Recent literature argues that investors require a return premium for an asset with such properties (e.g., Pástor and Stambaugh (2003), Acharya and Pedersen (2005), Sadka (2006), and Bekaert, Harvey, and Lundblad (2007)). To quantify this liquidity risk premium, we use the four-factor model of Pástor and Stambaugh (2003), which augments the Fama and French (1993) three-factor model with a traded liquidity factor.

As private equity investments are not traded, we do not have a time-series of returns and cannot apply the standard approach, which consists in projecting returns onto risk factors. As in Cochrane (2005), we exploit variation in returns across investments to estimate the risk loadings and abnormal performance of the asset class.

Based on the four-factor model, we find an alpha of zero, a liquidity beta of 0.7, a market beta of 1.3, and a positive and significant exposure to the book-to-market factor (HML). Using the average realization of the risk premiums in our sample as expected risk premiums, we infer that the historical cost of capital is around 24%. This cost of capital can be decomposed into the market risk premium (about 10%), the liquidity risk premium (about 3%), the value premium (about 5%), and the risk-free rate (about 6%, in our time period).[2]

Our paper contributes to two main strands of literature. On the one hand, we connect to the literature that relates liquidity risk to asset prices. A number of papers provide theoretical arguments for why investors want to be compensated for liquidity risk (e.g. Acharya and Pedersen (2005), Brunnermeier and Pedersen (2009), Chien and Lustig (2009), Chordia, Huh, and Subrahmanyam (2009)). The corresponding empirical literature emphasizes the relevance of systematic liquidity risk for public equity markets (e.g., Amihud (2002), Pástor and Stambaugh (2003), Acharya and Pedersen (2005), Sadka (2006) and Bekaert, Harvey, and Lundblad (2007)), bond markets (Beber, Brandt, and Kavajecz (2008), Chordia, Sarkar, and Subrahmanyam (2005) and Li, Wang, Wu, and He (2009)), credit derivative markets (Longstaff, Mithal, and Neis (2005), Bongaerts, de Jong, and Driessen (2008) and Longstaff, Pan, Pedersen, and Singleton (2007)), and hedge funds (Sadka (2009)).

On the other hand, we relate to the literature on risk and return of private equity investments (e.g. Moskowitz and Vissing-Jorgensen (2002) , Kaplan and Schoar (2005), Lerner, Schoar, and Wongsunwai (2007), Jones and Rhodes-Kropf (2004), Ljungqvist, Richardson, and Wolfenzon (2008), Hochberg, Ljungqvist, and Lu (2007)). Previous research finds that private equity funds underperform public equity after fees (Kaplan and Schoar (2005), Phalippou and Gottschalg (2009)). Here, we report evidence before fees from a different dataset. We show that performance before fees is consistent with previous research and that accounting for systematic risk brings alpha (gross of fees) close to zero.

This paper continues as follows. Section 2 describes the data and the liquidity measures. Section 3 relates private equity performance to liquidity risk. Section 4 computes the liquidity risk premium and the cost of capital for private equity. Section 5 discusses the implications of our results and concludes.

[2] Metrick (2007) measures liquidity risk for venture capital using the time-series of a venture capital index. He finds a 1% annual premium for venture capital liquidity risk.

2 The data

In this section, we first detail how the data are collected. Second, we document the coverage of our dataset over time (relative to available commercial datasets) and we gauge its representativeness in terms of performance.

2.1 Data source

The dataset is provided by the Center for Private Equity Research (CEPRES), a private consulting firm established in 2001 as a co-operation between the University of Frankfurt and Deutsche Bank Group. CEPRES' unique feature is that they collect information on the monthly cash flows generated by private equity investments.

CEPRES obtains data from private equity firms who make use of a service called "The Private Equity Analyzer".[3] Participating firms sign a contract that stipulates that they are giving the correct cash flows (before fees) generated for each investment they have made in the past. In return, the firm receives statistics such as risk-adjusted performance measures. These statistics are then used by the firm internally for various purposes like bonus payments or strengths/weaknesses analysis. Importantly, and unlike other data collectors, CEPRES does not benchmark private equity firms to peer groups. This improves data accuracy and representativeness as it eliminates incentives to manipulate cash flows or cherry-pick past investments. This program is very successful and, in 2009, it reached a coverage of 1200 private equity funds (including venture capital, buyout, infrastructure and mezzanine) with a geographical split of 50% from North America, 40% from Europe, and 10% from Asia.

In addition, CEPRES may be hired by investors as an advisor. CEPRES then receives data on the past performance of private equity firms in which these limited partners contemplate to invest. If the contractual agreement between the firm and the investor allows it, CEPRES can add that firm's (investment level) track record to their database. If the firm participates in the The Private Equity Analyzer program already, then CEPRES systematically cross-checks the data to verify that the above-mentioned contractual agreement

[3] In terms of vocabulary, private equity firms refers to organizations who run private equity funds, who in turn, make private equity investments in portfolio companies.

is respected.[4]

Earlier versions of this dataset have been utilized in previous studies. A subset of this database covering mainly venture capital investments is used by Cumming, Schmidt, and Walz (2009), Cumming and Walz (2009) and Krohmer, Lauterbach, and Calanog (2009). For this study, CEPRES granted us access to all liquidated buyout investments in their database as of December 2007. The earliest investment in our sample starts in 1975 and the last one in 2006. In total, we have 4,403 investments.

We thus have access to a comprehensive and accurate panel of cash-flow streams generated by private equity investments. This unique feature enables us to construct precise measures of the investment performance and aggregate liquidity conditions over the investment life, which is essential for estimating the relation between performance and liquidity risk.[5]

2.2 Data coverage and representativeness

We now compare our dataset to the two available commercial datasets most frequently used in academic research on private equity. We first document the coverage of our dataset over time and then its representativeness in terms of performance.

In terms of coverage, we can compare CEPRES to the (i) Standard & Poor's Capital IQ database at the investment level; and to (ii) Thomson Venture Economics (TVE) at the private equity firm level. These two benchmark datasets are perceived as the most comprehensive commercially-available datasets in private equity.

Table 1 – Panel A shows that CEPRES has a total of 7,198 private equity investments between 1975 and 2006 versus 18,934 in Capital IQ, or 38%.[6] From 1975 to 1994, the coverage of CEPRES and Capital IQ is remarkably similar. After the mid 1990s, the number

[4] A violation of this agreement has been spotted only once. The firm then corrected the entries shortly afterwards.

[5] Two proprietary databases are close to that of CEPRES and are used in contemporary research. Ljungqvist, Richardson and Wolfenzon (2008) have data from a large investor. Our data spans a similar time period as theirs, but contains about twice as many investments. Lopez-de-Silanes, Phalippou, and Gottschalg (2009) have a dataset with the performance of private equity investments from hand-collected private placement memoranda, but do not have the detailed cash flows generated for each investment.

[6] CEPRES defines private equity as: Acquisition financing, Leveraged Buy-Outs (LBO), Management Buy-Outs and Buy-Ins (MBO/MBI), Growth, Recapitalisation, Spin off, and Turnaround.

of investments in Capital IQ increases exponentially while the rise is less pronounced in CEPRES. Hence, coverage in CEPRES goes down but remains significant. In addition, as mentioned above, CEPRES provided us only with data on liquidated investments. Hence most of our data are from 1975 to 1999, a period during which CEPRES coverage is rather high (contains 63% of the observations in Capital IQ).

[Insert Table 1 about here]

Table 1 – Panel B shows results when we conduct a similar exercise with Thomson Venture Economics (TVE) dataset. TVE is the most frequently used dataset in academic research on private equity. In this literature, when return data is missing, performance is proxied by the success ratio (the fraction of investments exited via IPO or M&A over total number of investments). Hence, we compute the success ratio of the 117 firms that are in the CEPRES database and the 535 firms that are in TVE but not in the CEPRES database.[7] We find that the success ratio is similarly distributed for the two data sets, though CEPRES firms are slightly more successful on average.

We thus find that the distribution of performance in CEPRES seems representative. For instance, our fraction of bankrupted investments (10%) is the same as that reported by Strömberg (2007) for the Capital IQ dataset. Notice that, given that for this asset class returns are often observed only for IPO exited deals, our selection bias is relatively minor.

2.3 Performance Measures

As mentioned above, our data contain the series of cash flows generated by a given private equity investment. We begin by converting all the cash flows in US dollars. We note that this hardly changes performance. The correlation between performance in original currency and US dollars is 99.8%. This is probably because investments last only four years on average, hence currency changes do not affect performance much. In addition, about half of the cash-flows are already in US dollar.

To measure investment performance, we use a Modified Internal Rate of Return (MIRR). MIRR measures the geometric average return of an investment under an assumption on the

[7] Because we do not have access to fund and firm names, nor to non-liquidated investments, the statistics in this section were prepared for us by CEPRES on our behalf.

reinvestment of intermediate cash flows. Our assumption is that investors deposit dividends (D_t) into, and draw their money for intermediate investments (I_t) from, an account that earns an interest rate x_t for each $t = 1, \ldots, T$, where T is the number of periods in the investment life. The investment MIRR is defined as follows

$$
\begin{aligned}
(1+MIRR)^T &= \frac{D_1 \prod_{t=1}^{T-1}(1+x_t) + D_2 \prod_{t=2}^{T-1}(1+x_t) + \cdots + D_{T-1}(1+x_{T-1}) + D_T}{I_0 + \frac{I_1}{(1+x_0)} + \frac{I_2}{\prod_{t=0}^{1}(1+x_t)} + \cdots + \frac{I_T}{\prod_{t=0}^{T-1}(1+x_t)}} \\
&= \frac{FV(Div, x_t)}{PV(Inv, x_t)}. \qquad (1)
\end{aligned}
$$

where $FV(\cdot, x_t)$ and $PV(\cdot, x_t)$ denote respectively the forward and present value of a stream of period cash flows computed using the discount rate x_t. When no cash is returned to investors (that is, the dividends are all zero), the MIRR equals -100%.

We now give a numerical example of MIRR construction and its sensitivity to the reinvestment assumption. For that we use a typical cash flow pattern, which is shown in Table 2.[8] First, we assume a constant reinvestment rate of 10% per year.

The present value of the investments is:

$$PV(Div, 10\%) = 100 + \frac{25}{1.10^{0.25}} = 124.4$$

The final value of the dividends when re-invested at 10% yearly:

$$FV\,(Inv, 10\%) = 20(1.10^{2.5}) + 20(1.10^{1.5}) + 20(1.10^{0.5}) + 200 = 269.4$$

[8] In our sample, the median number of investments (negative cash flows) is two. The first one corresponds to the start of the investment. The second investment happens 3 months after the start (median) and equals 25% of initial investment (median). The median number of dividends is 4 (positive cash flows). The median duration is 4 years. The median duration for intermediate dividends is 2.5 years and we simply set dividend number one and three one year away. All intermediary dividends taken together are 60% of the initial investment (median) and we assume they are all equal. The (median) final dividend is twice the initial investment.

The annualized MIRR is thus:

$$MIRR = \left(\frac{269.4}{124.4}\right)^{1/4} - 1 = 21.3\%$$

If we use 0% as reinvestment rate instead of 10%, the MIRR is 20%. Hence, the sensitivity of MIRR to the reinvestment assumption is rather minor in our data. This is because of the relatively short life of the investments and the relatively small size of intermediary cash flows. In the analysis that follows, we use the S&P 500 index as a reinvestment rate in our main specification. This reinvestment assumption should capture the fact that private equity investors tend to be well diversified. We have also computed the MIRR of each investment using the risk free rate as reinvestment rate and find a coefficient of correlation of 99% between the two MIRRs.

[Insert Table 2 about here]

2.4 Descriptive statistics - Performance

To provide some aggregate performance figures, we group investments by their starting year and countries of location. Next, we sum the cash flows of all the investments in the group month by month. Finally, we compute the MIRR of these pooled cash flow streams. This measures the actual rate of return of a buy-and-hold investor who selects all the investments of a certain country/region and over a certain time period.

Table 3 shows the results. Overall, we find little difference across countries/regions and across time. Returns are highest for Europe (ex-UK) in the second half of the 1990s at 25% annually, but returns in Europe were low in the first half of the 1990s at 14% annually. Returns are stable over time in the US, except for more recent years where it drops to 13%. An investor buying all the investments in our sample would have earned 19% annually. The carried interest to be paid with such a return is about 4% and management fees on invested capital are at least 3%.[9] Hence, after fees, this performance figure is at most 12% yearly which is similar to the return documented by Kaplan and Schoar (2005) for net-of-fees (fund

[9] This is an approximation. Carried interest equals 20% times the MIRR. The 2% management fees is charged on a mix of capital invested and committed. Typically, this is equivalent to a 3% fee on capital invested (see Metrick and Yasuda, 2009, and Phalippou, 2009, for details)

level) returns. This further shows that our data are similar performance-wise to those used in previous research.

Table 3 also shows that our observations are almost equally distributed across the regions US (37%), UK (29%) and rest of Europe (25%).

3 Private equity performance and liquidity risk

In this section, we first emphasize that the focus of this paper is the relation between shocks to aggregate liquidity levels and private equity returns. That is, we focus on liquidity risk rather than liquidity level. Next, we describe our explanatory variables and, in particular, the measures of aggregate liquidity shocks. Finally, we perform a regression analysis of the determinants of private equity returns.

3.1 Liquidity risk

3.1.1 Liquidity risk and liquidity level

It is important to emphasize that the focus of this paper is not on the asset-specific liquidity characteristic (the liquidity level), but rather on the concept of market-wide liquidity as an non-diversifiable risk factor (the liquidity risk). In other words, our paper focuses on the compensation for systematic risk originating from time-varying liquidity.

The recent crisis illustrates these arguments and emphasizes how private equity may be particularly exposed to liquidity risk. Harvard University endowment has tried to sell a staggering $1.5 billion of private equity stakes in 2008 in an attempt to receive some cash from its private equity division at that time. It failed to sell this stake at a reasonable discount and as of mid-2009 has not sold it. Other private equity investors (e.g. pension funds) would have also probably preferred to receive large dividends from their private equity portfolio in 2008 rather than receiving them in 2006 (like they did).

The study of liquidity level in private equity is beyond the scope of this paper. It would include a study of the nascent secondary market for private equity stakes and the various restrictions private equity funds have for the transferability of these stakes. We do not have such data and the only study on this topic we are aware of is that of Lerner and Schoar (2004). They propose a model and provide supporting empirical evidence showing that the liquidity level of private equity funds is a decision variable for fund managers. Fund managers make the fund stakes illiquid on purpose and the degree to which they do it depends on the type of investments the fund makes.

3.1.2 Liquidity risk in private equity

Liquidity risk is likely to be of first order importance in private equity for a number of reasons. We discuss three channels in this sub-section.

Acharya and Pedersen (2005) argue that liquidity risk originates from uncertainty about the transaction costs faced when selling an asset. Let us consider a simple example with two equity positions A and B. A is a $10 million position in a S&P 500 company. B is a $10 million investment representing a 100% stake in a privately-held business.[10] Let us consider the uncertainty about the transaction costs that an investor will face when selling these positions. When selling A, a typical investor has considerably more flexibility to manage its transaction costs. For example, he can limit its price impact by splitting the order (Chan, Jegadeesh, and Lakonishok (1995), Vayanos (2001)). This option is not available for B. In addition, the depth of the market is probably better for A. The number of potential investors in A certainly varies over time, but we can assume that there will always be sufficient investors willing to purchase a $10 million stake in a S&P 500 company at a reasonable price. In contrast, the market for full ownership of a privately held business is significantly smaller. To exit a privately held investment, the two main routes are a trade sale or an IPO. Both of these exit channels have proven to be quite cyclical in the past. When exiting at a trough of the IPO or M&A cycle, the transaction costs are probably substantial. In contrast, in a buoyant IPO or M&A market, transaction costs are minimal. What this example suggests is that liquidity risk for a privately held company seems larger than for public equity, all else equal.

Another channel is investor's horizon. Private equity investors probably have longer

[10] The median equity size in our sample is $7 million and the mean is $19 million.

horizons and so have higher tolerance for liquidity risk than public equity investors.[11] Catering to these investors, a private equity fund would tilt its portfolio towards companies with high liquidity risk. As a result, private equity funds would hold a disproportionate fraction of high liquidity risk companies and their investors would earn the liquidity risk premium. Hence, factor loadings on liquidity risk may be higher for a private equity investment than for the average publicly traded stock. In addition, the existing literature is consistent with this view. Lerner and Schoar (2004) argue that private equity funds voluntarily put withdrawal restrictions in order to screen for investors that have a low chance of facing a liquidity shock. Hence, in their model, private equity investors have more tolerance for liquidity risk than the average investor. In addition, using a survey, Cumming and Johan (2009) show that investors rank liquidity among the main impediment for investing in private equity. Although the question does not distinguish between liquidity level and liquidity risk, this is an additional indication that investors with low tolerance for liquidity risk stay out of this asset class.

A third channel is that of leverage. Private equity companies are characterized by higher leverage than public equity companies.[12] As a result, they need to be refinanced more often. Providers of debt in private equity – mainly banks and hedge funds – are sensitive to funding liquidity. Brunnermeier and Pedersen (2009) provide a model that shows a positive link between an asset's "market liquidity" and traders' funding liquidity. Hence, times of low aggregate market liquidity coincide with times of low funding liquidity for traders such as hedge funds. Hedge funds in turn provide debt financing to the companies held by private equity firms. Hence when market liquidity is hit by a negative shock, private equity firms may find it difficult to refinance their companies and may be forced to liquidate a disproportionate number of their investments. Hence, all else equal, returns in private equity would be low when aggregate market liquidity is hit by a negative shock.

[11] The typical time horizon of the investment contract between a limited partner and a private equity fund is ten years.

[12] Kaplan and Strömberg (2009) argue that unlike public firms, private equity firms are aggressively taking advantage of debt mispricing in credit markets by increasing leverage. They also have better access to credit markets in general, because they are repeat borrowers, hence they are more prone to overleverage in cyclical waves. Axelson, Strömberg, and Weisbach (2009) provide a theoretical explanation for this large cyclicality and high leverage of private equity investments.

In a statement that provides support for the refinancing risk channel for private equity, Acharya, Philippon, Richardson, and Roubini (2009, p.9) argue that as a consequence of the recent liquidity crisis "[...] a large number of leveraged loans that are coming to maturity in 2010 and 2011 [...] may go bust once the refinancing crisis emerges". Also, a recent study by Standard & Poors (2009) corroborates this prediction and shows that defaults have already increased substantially due to covenant breaches.

3.2 Explanatory variables

3.2.1 Liquidity measures

The literature proposes several measures of shocks to aggregate (market-wide) liquidity level. We use three measures that are publicly available. First, we use the innovation in the aggregate liquidity measure of Pástor and Stambaugh (2003), which we denote "P&S LIQ innovation". Pástor and Stambaugh (2003) measure market-wide liquidity from the aggregation of firm-level OLS slopes of daily returns on signed daily trading volume within a month. Our second measure is the innovation in market illiquidity as computed by Acharya and Pedersen (2005), where the stock-level illiquidity is measured by the ratio of Amihud (2002). We multiply this measure by minus one to obtain a liquidity measure and denote it "A&P LIQ innovation". The third measure is that of Sadka (2006), denoted "Sadka LIQ innovation". Sadka (2006) proposes a measure of market-wide price impact, which he decomposes in a permanent and a transitory part. The permanent component is the one that according to Sadka is priced in public equity and it is thus the one that we use here.

Next, we compute the average liquidity shock during the life of each investment. This measures whether market-wide liquidity was surprisingly high or low between the starting date of the investment and the exit date of the investment. In the text below we refer to this variable as the 'liquidity conditions' faced by an investment.

Table 4 shows the correlation between the three measures of liquidity conditions. The correlation between the Pastor-Stambaugh and Acharya-Pedersen measures is 53%. The correlation of the Sadka measure with the other two measures is in both cases around 30%. The different measures of liquidity conditions are positively correlated, although not perfectly. Hence, in the analysis below, we show separate results for each measure.

[Insert Table 4 about here]

3.2.2 Control variables for the base specification

Amihud (2002) shows that aggregate liquidity shocks are positively correlated to stock-market returns. The average stock-market return during the life of an investment is basically the change in equity valuation between entry and exit. Hence, the average stock-market return during the life of an investment should be related to both investment performance and liquidity conditions. It is therefore an important control variable.

Consistent with this finding, we find that the average stock-market return, measured by the CRSP value-weighted index, is positively correlated with aggregate liquidity shocks. Table 4 shows that the coefficient of correlation is as high as 48% with the Pastor-Stambaugh measure and about 25% for the other two measures.

In addition, our dataset provides a number of investment and firm characteristics. We have information on investment size (in January-2007 USD), firm age, investment country of location, the industry of the investment, and the 'stage' of the investment (growth or LBO/MBO/MBI).

We control for the country and industry of the investment by adding corresponding fixed effects. Stage is added as a dummy variable that takes the value of one if the investment is classified as 'growth' and zero otherwise. As firm age and investment size increase over time, we subtract the annual mean to each observation. In addition, as there are some outliers in terms of size, we winsorize this variable at the 95th percentile.

3.3 Regression analysis

In this sub-section, we run OLS regressions with investment performance as the dependent variable. We first show results with the explanatory variables described in the previous sub-section. Second, we show results with cross-effects of liquidity and the main control variables. Third, we introduce macroeconomic variables as additional control variables.

3.3.1 Base specification

In the regressions that follow, the dependent variable is the investment MIRR (with S&P 500 as the reinvestment rate).[13] As we observe a fat right-hand-side tail, we winsorize MIRR at the 95th percentile and obtain a rather symmetric distribution (see Figure 1).[14] We also observe that 10% of the investments have an MIRR of -100%, which means that these investments returned no capital at all.

[Insert Figure 1 about here]

We use the z-score for each explanatory variables (except dummy variables). This way, each coefficient can be interpreted as the impact that a one-standard-deviation change in the explanatory variable has on annual returns. Inference is not affected by the transformation. Standard errors are clustered at the investment starting year because the performance of investments starting at the same time may be related.

Table 5 reports the results. Panel A focuses on the Pastor-Stambaugh measure of liquidity, Panel B on that of Acharya-Pedersen, and Panel C on that of Sadka.

The first specification shows the effect of liquidity conditions without any control variabes. The effect is economically and statistically significant. A one standard-deviation increase in liquidity conditions raises the annual MIRR by 13% for the Pastor-Stambaugh measure and by 7% for the other two measures.

The following specification adds the contemporaneous market return. As expected from the above discussion, this control reduces the slope on liquidity because part of the liquidity effect is reflected in changes in aggregate stock-market valuation (i.e. stock-market return). Magnitude varies across measures. The decrease is one quarter for the Pastor-Stambaugh measure, one third for the Acharya-Pedersen measure, and almost one half for the Sadka measure.

Adding further controls does not affect the coefficient on liquidity. The only control that turns out to be significant is the dummy variable for growth investments.[15] Next, we

[13] Note that in non-tabulated results, we repeat the analysis with the risk-free rate as the reinvestment rate (instead of the S&P 500) for the computation of MIRR. As the correlation between these two MIRRs is 99%, the results are virtually the same and are thus omitted.

[14] The 95th percentile is 135% annually and the 99th percentile is 400% annually.

[15] Growth investments underperform by 15% yearly. Part of the difference in returns could come from

add country and industry fixed effects. As we do not have information on country and industry for all observations, the sample size decreases slightly in these specifications, but the liquidity effect remains significant. We also note that for the country and industry fixed effects, no particular pattern emerges.[16]

[Insert Table 5 about here]

Providing a graphical impression of the economic magnitudes, Figure 2 shows the average MIRR by deciles of "liquidity conditions". The relation is monotone for the Pastor-Stambaugh measure. It moves from -13% average annual MIRR up to 33% average annual MIRR. Hence, the difference in performance between investments going through bad versus good liquidity conditions is as large as 46% per year. With the other measures, although the relation is not exactly monotone, the difference between top and bottom decile is also large. For the Sadka measure, the magnitude is similar to the Pastor-Stambaugh measure (the spread is 45%). For the Acharya-Pedersen measure the difference is somewhat smaller at 15% per year. Irrespective of the liquidity measure, investments that went through the most severe liquidity shocks (lowest decile) display negative performance, on average. In contrast, investments that went through the best liquidity times (e.g. the upper three deciles) return 20% per year on average, for each liquidity measure.

[Insert Figure 2 about here]

3.3.2 Investment characteristics and exposure to liquidity risk

We now investigate whether liquidity conditions have a stronger impact on certain types of investments. For this purpose, we interact the liquidity condition variable with (i) investment size, (ii) firm age, and (iii) the dummy variable for growth investments. We label these interactions "cross-effects".

We conjecture that larger investments are more sensitive to aggregate liquidity shocks. Transaction costs may be increasing and convex in size as in public equity (James and

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difference in leverage since growth investments use less leverage. We do not have the data to verify this and we are not aware of other studies documenting the performance of growth versus leveraged buyout investments.

[16] Partial exceptions are the significant underperformance of Israel and Spain and the significant outperformance of Russia.

Lipson (2001)). Intuitively, aggregate liquidity conditions may matter more when selling a large company than when selling a small company.

In addition, investment size may be positively related to financial leverage as it is the case for public equity. Consistent with this conjecture, a recent Moody's research report points out that in the wake of the recent financial crisis, the worst performing deals are among the largest ones. According to the report, "[...] it appears that when you do a large dollar value transaction and you lever that company up, you seem to be at more risk of having problems in a downturn".[17] Hence the leverage channel exposed in section 3.1.2 also suggests that larger investments may be more sensitive to liquidity conditions.

Results in Table 6 confirms this conjecture. Larger investments are significantly more sensitive to liquidity conditions based on the Acharya-Pedersen and Sadka measures. For the Pastor-Stambaugh measure, the relation is not always significant.

We also conjecture that more experienced private equity firms may invest in companies that are more sensitive to aggregate liquidity shocks. Older firms may have investors with deeper pockets and thus load more on liquidity risk. In addition, older firms are typically more levered (James and Demiroglu (2009)). It is also the case that older firms pursue larger investments and we should see whether the size effect we just documented is dominated by 'firm age' or not.

Results show that older firms hold investments that are more sensitive to liquidity conditions. The effect is always statistically significant for both the Pastor-Stambaugh and the Acharya-Pedersen measures. It is positive but not significant for the Sadka measure. The firm-age cross-effect, however, does not drive out the investment-size cross-effect.

Finally, we find that growth investments do not have a significantly different exposure to liquidity shocks.

[Insert Table 6 about here]

[17] The study we refer to is "$640 Billion & 640 Days Later: How Companies Sponsored by Big Private Equity Have Performed During the U.S. Recession." The lead author for the report is John Rogers, a senior vice president at Moody's. We currently do not have direct access to the study, but the study is summarized in a New York Times article by Jenny Anderson (2009).

3.3.3 Macroeconomic conditions and exposure to liquidity risk

Liquidity conditions are certainly related to a number of macroeconomic variables. The above effect may thus be a manifestation of a good 'macroeconomic' environment fostering positive private equity returns. Chen, Roll, and Ross (1986) show that two macro factors are priced in the cross section of stocks. These factors are default spread and growth in industrial production. According to these authors, the default spread proxies for changes in the expected risk premium and industrial production growth proxies for changes in future profitability.

In Table 7, we use these two variables as controls. The default spread is the difference in yields between BAA and AAA rated U.S. bonds. We use the U.S. measure of industrial production. Both variables are taken from the Federal Reserve at Saint Louis website.[18] We find that average default spreads during investment life are not related to private equity returns. In contrast, growth in industrial production is strongly related to private equity returns. The effect of growth in industrial production is similar in magnitude to that of contemporaneous stock-market returns and liquidity conditions. Adding this control variable, however, does not weaken the relation between performance and liquidity conditions. If anything, it strengthens it.

[Insert Table 7 about here]

Cumming, Flemming, and Schwienbacher (2005) argue that IPO volume is positively correlated with aggregate liquidity in venture capital. In private equity, the state of the IPO market during the investment life may also be related to both performance and liquidity. For IPO volume, we use the number of U.S. IPOs from Prof. Jay Ritter's website.[19] Table 4 shows that innovations in IPO volume are positively correlated with aggregate liquidity shocks irrespective of the liquidity measure used. The correlations range between 12% and 45%. Innovations in IPO volume are also positively correlated with contemporaneous market returns, strongly negatively correlated with average default spreads, and strongly positively correlated with average industrial production growth.

Table 7 shows that innovations in IPO volume during the investment life are not related

[18] http://research.stlouisfed.org/fred2/series/INDPRO.
[19] http://bear.warrington.ufl.edu/Ritter/ipodata.htm.

to private equity returns once we control for stock-market returns and liquidity.[20] Without these controls, however, changes in IPO volume are positively related to returns.

Spiegel and Wang (2005) and Bandi, Moise, and Russel (2008) argue that the effects on returns of aggregate liquidity and aggregate volatility are closely related. Consistent with their argument that liquidity improves when volatility goes down, we observe a negative correlation between innovations in volatility (proxied by the VIX index) and both the Acharya-Pedersen liquidity innovations and the Sadka liquidity innovations (see Table 4).[21] However, the correlation is zero with the Pastor-Stambaugh measure.

In Table 7, for all liquidity measures, we observe no relation between innovations in aggregate volatility and private equity performance. In addition, controlling for volatility shocks does not affect the relation between liquidity and performance.

4 Private equity liquidity risk premium and cost of capital

4.1 Methodology

In this section, we develop our econometric approach to estimate the cost of capital for private equity investments.

To start from the simplest case, let us assume that the cash flows of each project i consist of an initial investment, V_0^i, and a final dividend, $V_{T_i}^i$, which is paid at date T_i.

Similar to Cochrane (2005), we assume that one-period returns are log-normal and exhibit a factor structure (in logarithm)

$$\ln R_{t+1}^i = \ln \frac{V_{t+1}^i}{V_t^i} = \gamma + \ln R_{t+1}^f + \delta' f_{t+1} + \varepsilon_{t+1}^i \tag{2}$$

where γ is a constant, R^f is the gross risk free rate, f_{t+1} is a vector of k risk factors, δ is a k-vector of risk factor loadings, and ε_{t+1}^i is normal with mean zero and variance σ^2 and is independent of the risk factors. Given the monthly frequency of the factors, we set

[20] Innovations are those from an AR(1) process estimate over the full sample period. Results are similar when we use IPO volume level instead of innovations.

[21] The CBOE Volatility Index (VIX) is a measure of market expectations of near-term volatility conveyed by S&P 500 stock index option prices. Innovations in volatility is simply the change in VIX from one month to the other. Fitting an AR(1) leads to similar results.

the interval length to one month. This choice has no material consequences, except for the interpretation of the reported coefficients.

Notice that, unlike Cochrane (2005), we choose to have the normally distributed factors in levels rather than logs. The reason is that, in our multifactor framework, some factors are based on long-short strategies and can take negative values, which does not allow the logarithmic transformation. This fact causes minor deviations from Cochrane in the formulas for factor loadings and alphas that we derive in the Appendix.

Given equation (2), the natural logarithm of the (gross) geometric average return on the investment (R_g^i) is given by

$$\ln(R_g^i) = \frac{1}{T_i} \ln \frac{V_T^i}{V_0^i} = \gamma + \frac{1}{T_i} \sum^{T_i} \ln R_{t+1}^f + \delta' \frac{1}{T_i} \sum^{T_i} f_{t+1} + \frac{1}{T_i} \sum^{T_i} \varepsilon_{t+1}^i. \tag{3}$$

The variance of the error term in equation (3) is $\frac{1}{T_i}\sigma^2$. To eliminate this source of heteroskedasticity, we multiply each side of Equation (3) by $\sqrt{T_i}$

$$\sqrt{T_i} \ln(R_g^i) = \gamma \sqrt{T_i} + \frac{1}{\sqrt{T_i}} \sum^{T_i} \ln R_{t+1}^f + \delta' \frac{1}{\sqrt{T_i}} \sum^{T_i} f_{t+1} + \frac{1}{\sqrt{T_i}} \sum^{T_i} \varepsilon_{t+1}^i. \tag{4}$$

This is the specification that we bring to the data.

This approach is motivated by the specific structure of the data. We do not observe periodic valuations of the investment. Hence, we cannot construct a time-series of investment returns R_{t+1}^i. Instead, we observe the investment cash flows, which allow us to construct a summary measure of performance over the investment life, that is, the geometric average return R_g^i. This explains why, in equation (4), we relate the average investment return to the average factor realization over the investment life (with a correction for heteroskedasticity).

In summary, our procedure boils down to considering each investment as a separate realization of returns on the asset class and exploiting variation in returns across investments to estimate the risk loadings and abnormal performance of the asset class. This cross-sectional framework to estimate factor loadings is in the same spirit as Cochrane (2005) and Kaplan and Strömberg (2009).

Because the parameters in equation (2) pertain to the logarithm of returns, we need to derive expressions for alpha and factor loadings for the level of returns. In other words, we need to convert (γ, δ) from equation (2) into (α, β) in the following standard factor representation of expected returns

$$E\left(R^i_{t+1}\right) = R^f_{t+1} + \alpha + \beta' E\left(f_{t+1}\right). \tag{5}$$

In the appendix, we show that the formulas for conversion from the log to the level of returns are

$$\beta = R_f \delta e^{\gamma + \delta' \mu_F + \frac{1}{2}\delta' \sigma^2_F \delta + \frac{1}{2}\sigma^2} \tag{6}$$

$$\alpha = R_f \left(e^{\gamma + \delta' \mu_F + \frac{1}{2}\delta' \sigma^2_F \delta + \frac{1}{2}\sigma^2}(1 - \delta\prime\mu_F) - 1\right) \tag{7}$$

where μ_F is the k-vector of factor means and σ^2_F is the $k \times k$ variance-covariance matrix of the factors. Because β turns out to be quite close to δ, we only report the latter.

Finally, we define the cost of capital as the risk free rate plus the estimate of each factor loading times the average realization of the corresponding factor in the sample. Based on equation (5), we note that the sum of alpha and the cost of capital gives an estimate of the investment expected return.

4.2 Estimation

As shown in Table 2, investments in our data may have intermediate cash flows. Hence, like in the analysis above, we use MIRR to measure the geometric return in equation (4).

One obstacle to the estimation of equation (4) is that the logarithm of MIRR is not defined for 10% of the investments (those with return of -100%). In addition, as for the estimation of risk loadings for stocks, the high idiosyncratic risk of individual investments may induce substantial noise in our estimates.

To get around these issues, we adopt the typical asset pricing strategy, which consists of grouping individual investments into portfolios. Because our identification comes from observing investment returns at different moments in time, a natural choice is to group together investments that start at the same date (at the monthly frequency). By grouping investments based on their starting dates, we reduce idiosyncratic risk and preserve sufficient dispersion in the explanatory variables.[22]

To assure that portfolios are sufficiently diversified, we require a minimum of thirty investments per portfolio. If the number of investments starting at the same month is below this minimum, we include in the portfolio investments that are started the next month and so forth until the number of investments is at least thirty.[23]

Portfolio cash flow streams are obtained by summing the cash flows of the individual investments each month. Next, we compute the MIRR of each portfolio. We do not observe any portfolios for which the MIRR is -100%. So, the logarithm of the MIRR is always defined. We can thus estimate equation (4) by OLS at the portfolio level.

4.3 Risk loadings, alpha, and the cost of capital

The estimate of the cost of capital is conditional on the choice of a specific asset pricing model. We provide results from three different models. We start with the CAPM, which is the model that Cochrane (2005) estimates for venture capital. Then, recognizing that private equity investments tend to be made predominantly in value companies, we consider the Fama and French (1993) three-factor model. Finally, following the above evidence that returns are sensitive to liquidity conditions, we augment the three-factor model with the Pástor and Stambaugh (2003) liquidity factor.[24] This factor corresponds to a long position in high-liquidity-beta stocks and a short position in low-liquidity-beta stocks.

Table 8 shows the correlation and distribution of the factors during our sample time period (October 1975 to December 2007). In particular, it shows the time-series mean of each factor. We use these means as estimates of the factor risk premiums in the computation of the cost of capital. Multiplying by twelve the values in Table 8, the liquidity premium is 4.5% annually. The market risk premium is 7.5% annually. The HML and SMB premiums

[22] At the other extreme, if the portfolio cash flows all started in 1975 and ended in 2007, there would be no variation in the explanatory variables (the risk factors), and we would not be able to identify the risk loadings.

[23] In the appendix tables, we show robustness to different choices for the minimum number of investment per portfolios.

[24] We use the original Pastor and Stambaugh (2003) measure which is based on predicted betas. The new version of this factor, which is posted on Lubos Pastor's website and is based on historical betas, produces quantitatively similar results.

are 4.9% and 2.9% annually, respectively. The (unreported) risk-free rate is 5.8% annually.

[Insert Table 8 about here]

Panel A of Table 9 reports the estimates of equation (4) for each of the factor models. The estimate of the CAPM beta is close to unity and highly significant. This number is consistent with the choice of Kaplan and Schoar (2005) to measure private equity performance by a public market equivalent with a beta of one.

The second column in Panel A reveals that, after accounting for the other Fama and French (1993) factors, the loading on the market increases by roughly 50%. This is due to the fact that private equity investments load positively (and significantly at the 10% level) on value stock returns and that HML and the market factor are negatively correlated in this sample. The loading on SMB is positive, but not statistically different from zero.

Finally, the last column in Panel A reports the estimates of the model with liquidity risk. Consistent with the results in the previous section, private equity returns load significantly on the liquidity factor. The liquidity beta is about 0.7. Relative to the three-factor model, the loading on the market decreases to 1.3. The slope on HML increases and becomes statistically significant at the 5% level, suggesting that in the previous model the importance of HML is mitigated by the negative correlation between HML and the liquidity factor.

For each of the factor models, Panel B of Table 9 reports the alpha and cost of capital estimates that result from transforming the model in logs to the model in levels according to equations (5) and (6). The cost of capital is simply the average risk free rate plus the products of factor loadings and factor risk premiums.

The CAPM alpha is about 8.5% annually. The cost of capital with this model is the lowest at roughly 13.5% per year. In the second column, once the risk premiums on the book-to-market and size factors are taken into account, the alpha drops to about 2.3% which is still economically (although not statistically) significant. With the Fama-French model, the cost of capital rises to around 20.8% annually and the premium for market risk accounts for more than half of this amount. In the model with liquidity risk (third column of Panel B), the premiums on the four factors entirely account for average private equity returns. The alpha is virtually zero, both economically and statistically, and the cost of capital is

23.8% per year. Of this amount, the lion's share belongs to the market risk premium (10%). Then, the book-to-market premium is 4.7% but the size premium is negligible. Finally, the liquidity risk premium amounts to a statistically and economically significant 3.1% per year. Hence, it appears that the liquidity risk premium is an important component to provide a full account of average private equity returns.

Our choice of a minimum number of thirty investments per portfolio is somewhat arbitrary. To address this concern, Tables A-I and A-II in the appendix report the results with alternative choices for this minimum threshold. The threshold cannot drop below five investments otherwise we have a portfolio with -100% return. With a threshold of fifty investments per portfolio, the number of portfolios drops to sixty-eight. So, we do not go further. Results appear remarkably stable throughout the spectrum of thresholds.

To conclude, notice that by adding the estimates of alpha and the cost of capital in Panel of Table 9 one obtains an estimate of the expected return of private equity of about 24% (for the four-factor model). This estimate is larger than the 19% average return that we report in Panel A of Table 3. This spread is due to the fact that Table 3 has the average geometric return (R_g^i), whereas the 24% estimate refers to arithmetic returns (R_{t+1}^i). Whenever the volatility of returns is not zero, the geometric return is smaller than the arithmetic return.

[Insert Table 9 about here]

5 Conclusions

Using cash flows from 4,403 liquidated private equity investments from a novel and comprehensive dataset, we find a positive and significant relation between returns and liquidity risk measures. These results are robust to the use of different measures for liquidity risk. They also survive after controlling for macroeconomic conditions and investment characteristics. Larger investments have higher exposure to liquidity conditions and so do investments made by older private equity firms. These findings are consistent with three economic channels for an effect of liquidity risk on private equity returns: transaction costs, risk tolerance of private equity investors, and leverage. Depending on data availability, future research may further investigate the separate contribution of these channels. Finally, using the Pástor and

Stambaugh (2003) four-factor model, we find that the liquidity risk premium is about 3% annually, the cost of capital is about 24%, and the alpha (gross-of-fees) is not statistically different from zero.

The results in this paper are relevant for academics and practitioners alike as we quantify the systematic risks and the efficiency of the pricing of an asset class that has gained increasing weight in financial markets. Our evidence suggests that the apparently high performance (before fees) of private equity investments can be explained as a compensation for the different risk factors to which returns are exposed and, in particular, to liquidity risk. Moreover, the risk profile of private equity is an important input for the risk management of large investors. In particular, Basel II imposes banks a certain provision for the risk of their private equity investments (see Bongaerts and Chalier (2009)). As these calculations may not account for the large exposure to liquidity risk, they may imply too low a provision.

Related to this, our results suggest that the increased allocation to private equity may lengthen financial crises. As the recent financial crisis illustrates, liquidity (or lack of) is often a major driver of financial crises (see Brunnermeier and Pedersen (2009) for an overview). If private equity investors have particularly large losses during times of low liquidity, these losses will materialize at a slow pace for a prolonged period of time because of the long span of a private equity investment. This, in turn, can maintain financial markets under stress for longer than they would otherwise.

Finally, this paper finds that the same liquidity risk factor identified in public equity is consistently related to private equity performance. This contributes to a recent literature showing the pervasiveness of liquidity risk across asset classes.

Appendix

In this appendix, we provide the explicit derivation of equations (7) and (6) in the text. The reported formulas differ slightly from the ones in Cochrane (2005), because we have a multifactor model and the factors are not in logarithm.

To compute beta we use the standard definition

$$\beta = Var\left(f_{t+1}\right)^{-1} Cov\left(f_{t+1}, R^i_{t+1}\right). \tag{A-1}$$

From equation (2), R^i_{t+1} is the exponential of a normally distributed variable:

$$R^i_{t+1} = R^f_{t+1} e^{\gamma+\delta' f_{t+1}+\varepsilon^i_{t+1}}$$

Also, by assumption, the factors are normal. Hence, the expression of the expected return is

$$E\left(R^i_{t+1}\right) = R^f_{t+1} e^{\gamma+\delta'\mu_F+\frac{1}{2}\delta'\sigma^2_F\delta+\frac{1}{2}\sigma^2} \tag{A-2}$$

Applying Stein's lemma, the covariance can be expressed as

$$\begin{aligned} Cov\left(f_{t+1}, R^i_{t+1}\right) &= Cov(f_{t+1}, \delta' f_{t+1} + \varepsilon^i_{t+1}) E\left(R^i_{t+1}\right) \\ &= Cov\left(f_{t+1}, \delta' f_{t+1} + \varepsilon^i_{t+1}\right) R^f_{t+1} e^{\gamma+\delta'\mu_F+\frac{1}{2}\delta'\sigma^2_F\delta+\frac{1}{2}\sigma^2} \\ &= Var\left(f_{t+1}\right) \delta R^f_{t+1} e^{\gamma+\delta'\mu_F+\frac{1}{2}\delta'\sigma^2_F\delta+\frac{1}{2}\sigma^2} \end{aligned}$$

where, for the last step, we use the fact that ε^i_{t+1} and f_{t+1} are uncorrelated. The expression for beta then follows:

$$\begin{aligned} \beta &= Var\left(f_{t+1}\right)^{-1} Var\left(f_{t+1}\right) \delta R^f_{t+1} e^{\gamma+\delta'\mu_F+\frac{1}{2}\delta'\sigma^2_F\delta+\frac{1}{2}\sigma^2} \\ &= \delta R^f_{t+1} e^{\gamma+\delta'\mu_F+\frac{1}{2}\delta'\sigma^2_F\delta+\frac{1}{2}\sigma^2} \end{aligned} \tag{A-3}$$

To compute alpha we use the standard definition

$$\alpha = E\left(R^i_{t+1}\right) - R^f_{t+1} - \beta' E\left(f_{t+1}\right) \tag{A-4}$$

where $E\left(f_{t+1}\right) = \mu_f$. Replacing the expressions for the expected return in (A-2) and beta in (A-3), we get

$$\alpha = R_f \left(e^{\gamma + \delta' \mu_F + \frac{1}{2}\delta' \sigma_F^2 \delta + \frac{1}{2}\sigma^2} (1 - \delta\prime \mu_F) - 1 \right) \tag{A-5}$$

Although we do not use them in the estimation, it is interesting to derive the continuous time limits for α and β. These are:

$$\beta = \delta \tag{A-6}$$

$$\alpha = \gamma + \frac{1}{2}\delta' \sigma_f^2 \delta + \frac{1}{2}\sigma^2. \tag{A-7}$$

To obtain these formulas, one can start from the continuous time equivalent of equation (2)

$$d\log\left(V_t\right) = \gamma dt + r_f dt + \delta' df_t + \sigma dZ_t \tag{A-8}$$

where $df_t = \mu_f dt + \sigma_f dZ_{f,t}$, Z_t and $Z_{f,t}$ are independent vectors of standard Brownian motions, and r_f is the instantaneous risk free rate. Then, apply Ito's lemma to equation (A-8) to get the process for the return in levels

$$\begin{aligned} \frac{dV_t}{V_t} &= \left(\gamma + r_f + \delta' \mu_f + \frac{1}{2}\left(\sigma^2 + \delta' \sigma_f^2 \delta\right) \right) dt \\ &\quad + \sigma dZ_t + \delta' \sigma_f dZ_f. \end{aligned} \tag{A-9}$$

Then, from equation (A-9), we obtain beta using the standard definition

$$\begin{aligned} \beta &= Var\left(df_t\right)^{-1} Cov\left(df_t, \frac{dV_t}{V_t}\right) \\ &= \left(\sigma_f^2\right)^{-1} \sigma_f^2 \delta \\ &= \delta. \end{aligned} \tag{A-10}$$

Finally, to obtain equation (A-7), use the standard definition of alpha and the result in (A-10)

$$\begin{aligned}
\alpha dt &= E\left(\frac{dV_t}{V_t}\right) - r_f dt - \beta' E\left(df_t\right) \\
&= \left(\gamma + r_f + \delta'\mu_f + \frac{1}{2}\left(\sigma^2 + \delta'\sigma_f^2\delta\right)\right) dt - r_f dt - \delta'\mu_f dt \\
&= \left(\gamma + \frac{1}{2}\left(\sigma^2 + \delta'\sigma_f^2\delta\right)\right) dt.
\end{aligned}$$

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Table 1: Sample selection and performance. The table reports coverage of the CEPRES dataset at the investment level (compared to Capital IQ, Panel A) and at the Private Equity firm level (compared to Thomson Venture Economics, Panel B).
In Panel B, successful exit rate is the number of investments exited by IPO or M&A divided by the total number of investments of a given firm. Only investments made before 2002 are included and a minimum of 5 investments is required.

Panel A: Investment coverage (CEPRES versus Capital_IQ)

	Liquidated			Non-liquidated			Total CEPRES	Capital IQ	Fraction
	Number Investments	Size ($ million)	Multiple Mean (VW)	Number Investments	Size ($ million)	Multiple Mean (VW)	Number Investments	Number Investments	CEPRES vs Capital IQ
1975-1989	592	4,581	3.30	10	237	3.56	602	688	0.88
1990-1994	1,320	17,354	3.03	52	1,523	2.20	1,372	1,550	0.89
1995-1999	1,702	38,059	2.38	665	28,569	1.65	2,367	4,660	0.51
2000-2006	789	27,027	2.26	2,068	140,003	1.66	2,857	12,036	0.24
Total	4,403	87,021	2.52	2,795	170,332	1.67	7,198	18,934	0.38

Table 1: (continued)

Panel B: Firm coverage (CEPRES versus Thomson Venture Economics)

	CEPRES & TVE (1)	TVE only (2)	Difference (1)-(2)
Successful exit			
Number of firms	117	535	-418
20th percentile	0.43	0.39	0.04
50th percentile	0.61	0.56	0.05
80th percentile	0.75	0.72	0.03
Mean	0.59	0.55	0.04

Table 2: Cash flows of a typical investment. A typical cash flow stream is shown. The initial investment is normalized to 100. To construct it, we first compute the median number of intermediate cash inflows and outflows. The result is 1 and 3, respectively. Next, we compute the median time at which intermediate cash inflows and outflows occur. The result is 0.25 and 2.5 years respectively. Next, we compute the median size of intermediate cash inflows and outflows (normalized to initial amount invested). The result is 25% and 195% respectively. Then we compute the median size of the final cash outflow and its median time. The result is 4 years and 135%.

Date (in years)	Cash flows
0.00	-100
0.25	-25
0.50	0
0.75	0
1.00	0
1.25	0
1.50	20
1.75	0
2.00	0
2.25	0
2.50	20
2.75	0
3.00	0
3.25	0
3.50	20
3.75	0
4.00	200

Table 3: Performance by year and region. The table reports Modified IRR of investments started in different years and regions. The reinvestment rate is the return on the S&P 500 index. Performance is computed on the pooled cash flows.

Panel A: Modified Internal Rates of Return (S&P as re-investment rate)

	1975-1989	1990-1994	1995-1999	2000-2006	1975-2006
US	0.18	0.18	0.19	0.13	0.18
UK	0.17	0.16	0.17	0.20	0.17
Europe (ex-UK)	0.17	0.14	0.25	0.21	0.20
Rest world	0.21	0.15	0.18	0.17	0.17
All countries	0.18	0.17	0.21	0.21	0.19

Panel B: Number of Investments

	1975-1989	1990-1994	1995-1999	2000-2006	1975-2006
US	323	533	534	237	1627
UK	172	440	526	139	1277
Europe (ex-UK)	68	269	499	246	1082
Rest world	17	23	121	152	313
All countries	592	1320	1702	789	4403

Table 4: Correlations of the explanatory variables. The table reports the correlation matrix of investment return (Modified IRR with the S&P 500 index as reinvestment rate), investment liquidity condition measures, average excess stock-market return during investment life (Rm-Rf), and investment macro condition measures. Liquidity and macro condition measures are the average (equally weighted, monthly frequency) of the corresponding variable during investment life. There are three liquidity measures: Pastor-Stambaugh (2003) aggregate liquidity innovations, Acharya and Pedersen (2005) aggregate liquidity innovations, Sadka (2006) aggregate liquidity innovations. There are four macro measures: default spread, industrial production growth, innovation in the number of IPO (from an AR(1) model), change in the VIX index (month t minus month $t-1$). The correlations are computed across investments.

		Var 1	Var 2	Var 3	Var 4	Var 5	Var 6	Var 7	Var 8
Var 1	MIRR	1.00							
Var 2	P&S LIQ innovation	0.21							
Var 3	A&P LIQ innovation	0.12	0.53						
Var 4	Sadka LIQ innovation	0.11	0.32	0.28					
Var 5	Rm-Rf	0.19	0.48	0.24	0.27				
Var 6	Default spread	-0.08	-0.26	0.25	-0.11	-0.38			
Var 7	Industrial Production Growth	0.22	0.34	0.04	0.15	0.70	-0.59		
Var 8	IPO volume innovation	0.13	0.45	0.12	0.12	0.63	-0.62	0.68	
Var 9	VIX innovation	-0.01	0.00	-0.16	-0.24	-0.20	-0.37	0.25	0.37

Table 5: Base regression analysis. The table reports the results of OLS regressions. The dependent variable is the Modified IRR (S&P 500 index as reinvestment rate) of individual investments. Explanatory variables include a measure of aggregate liquidity innovations during the investment life, the average excess market return (CRSP value-weighted index minus the risk free rate) during investment life, investment size, firm age at the time of the investment, a dummy variable that takes the value one if the investment is classified as growth (zero otherwise). Size and firm age are relative to same-year investments and are winsorized (at 5th and 95th percentiles of same-year investments). Each explanatory variable is expressed as a z-score (subtract mean and divide by standard deviation for each variable). Country and industry fixed effects may be added to the set of explanatory variables. Standard errors are clustered at the investment year level. T-statistics are reported below the coefficients in parentheses. The liquidity variable is either that of Pastor and Stambaugh (Panel A), Acharya and Pedersen (Panel B) or Sadka (Panel C).

Panel A: P&S innovations

P&S LIQ innovation	0.126	0.092	0.092	0.092	0.095	0.095	0.097	0.098	0.100
	(4.410)	(3.361)	(3.342)	(3.357)	(3.547)	(3.537)	(3.695)	(3.511)	(3.690)
Rm-Rf		0.070	0.070	0.070	0.064	0.063	0.063	0.061	0.063
		(2.443)	(2.466)	(2.440)	(2.248)	(2.222)	(2.176)	(1.994)	(2.130)
Size			0.014			0.000	0.000	-0.004	-0.001
			(1.257)			(0.016)	(0.038)	(-0.327)	(-0.114)
Firm age				0.016		0.026	0.024	0.028	0.021
				(1.084)		(1.499)	(1.321)	(1.676)	(1.225)
Growth investment					-0.153	-0.162	-0.169	-0.164	-0.175
					(-3.778)	(-3.876)	(-3.998)	(-3.997)	(-4.176)
Country fixed effects	no	no	no	no	no	no	yes	no	yes
Industry fixed effects	no	no	no	no	no	no	no	yes	yes
Adj. R2	0.045	0.056	0.056	0.056	0.067	0.069	0.079	0.079	0.091
N	4403	4403	4403	4403	4403	4403	4275	4118	4037

Table 5: (continued)

Panel B: A&P innovations

A&P LIQ innovation	0.074	0.049	0.048	0.049	0.056	0.057	0.055	0.059	0.064
	(2.255)	(2.103)	(2.088)	(2.108)	(2.660)	(2.713)	(2.381)	(2.520)	(2.685)
Rm-Rf		0.102	0.103	0.102	0.096	0.095	0.097	0.094	0.097
		(4.087)	(4.124)	(4.076)	(3.905)	(3.870)	(4.006)	(3.567)	(3.819)
Size			0.014			-0.001	0.000	-0.004	-0.002
			(1.147)			(-0.117)	(-0.023)	(-0.372)	(-0.125)
Firm age				0.017		0.028	0.024	0.030	0.023
				(1.127)		(1.585)	(1.333)	(1.746)	(1.286)
Growth investment					-0.160	-0.170	-0.176	-0.173	-0.185
					(-3.873)	(-4.075)	(-4.170)	(-4.121)	(-4.288)
Country fixed effects	no	no	no	no	no	no	yes	no	yes
Industry fixed effects	no	no	no	no	no	no	no	yes	yes
Adj. R2	0.015	0.043	0.044	0.044	0.056	0.058	0.067	0.067	0.080
N	4403	4403	4403	4403	4403	4403	4275	4118	4037

Panel C: Sadka innovations

Sadka LIQ innovation	0.067	0.037	0.037	0.038	0.034	0.035	0.034	0.037	0.035
	(3.115)	(1.959)	(1.967)	(1.981)	(1.749)	(1.756)	(1.739)	(1.933)	(1.890)
Rm-Rf		0.105	0.105	0.105	0.101	0.101	0.101	0.099	0.102
		(3.708)	(3.722)	(3.699)	(3.629)	(3.573)	(3.604)	(3.227)	(3.447)
Size			0.005			-0.009	-0.009	-0.014	-0.011
			(0.401)			(-0.725)	(-0.605)	(-1.188)	(-0.751)
Firm age				0.012		0.025	0.021	0.027	0.021
				(0.756)		(1.353)	(1.150)	(1.565)	(1.150)
Growth investment					-0.151	-0.166	-0.172	-0.169	-0.179
					(-3.278)	(-3.550)	(-3.649)	(-3.584)	(-3.849)
Country fixed effects	no	no	no	no	no	no	yes	no	yes
Industry fixed effects	no	no	no	no	no	no	no	yes	yes
Adj. R2	0.012	0.043	0.043	0.043	0.053	0.055	0.065	0.065	0.075
N	4020	4020	4020	4020	4020	4020	3905	3748	3676

Table 6: Regressions with cross-effects. The table reports the results of OLS regressions. The dependent variable is the Modified IRR (S&P 500 index as reinvestment rate) of individual investments. Explanatory variables are the same as in Table 5 except for the cross effects. Each explanatory variable is expressed as a z-score (subtract mean and divide by standard deviation for each variable). Country and industry fixed effects may be added to the set of explanatory variables. Standard errors are clustered at the investment year level. T-statistics are reported below the coefficients in parentheses. The liquidity variable is either that of Pastor and Stambaugh (Panel A), Acharya and Pedersen (Panel B) or Sadka (Panel C).

Panel A: Pastor and Stambaugh liquidity measure

P&S LIQ innovation	0.096	0.097	0.080	0.082	0.086	0.081	0.085
	(3.776)	(3.863)	(2.594)	(2.950)	(3.062)	(2.836)	(2.947)
Rm-Rf	0.062	0.063	0.062	0.060	0.061	0.057	0.060
	(2.176)	(2.215)	(2.163)	(2.112)	(2.075)	(1.868)	(2.014)
Size	0.009	0.006	0.004	0.009	0.009	0.006	0.006
	(0.742)	(0.458)	(0.288)	(0.761)	(0.655)	(0.485)	(0.462)
Firm age	0.021	0.021	0.017	0.019	0.018	0.021	0.015
	(1.196)	(1.569)	(1.039)	(1.391)	(1.178)	(1.489)	(1.000)
Growth investment	-0.158	-0.166	-0.158	-0.165	-0.170	-0.165	-0.175
	(-3.882)	(-4.077)	(-3.606)	(-3.785)	(-3.832)	(-3.801)	(-4.008)
P&S LIQ innovation * Size	0.020			0.020	0.019	0.022	0.019
	(1.697)			(1.633)	(1.303)	(1.561)	(1.182)
P&S LIQ innovation * Firm age		0.034		0.029	0.027	0.026	0.023
		(2.673)		(2.415)	(2.330)	(2.105)	(1.882)
P&S LIQ innovation * Growth investment			0.062	0.068	0.061	0.078	0.071
			(1.583)	(1.810)	(1.692)	(2.077)	(1.917)
Country fixed effects	no	no	no	no	yes	no	yes
Industry fixed effects	no	no	no	no	no	yes	yes
Adj. R2	0.070	0.072	0.071	0.075	0.084	0.086	0.096
N	4403	4403	4403	4403	4275	4118	4037

Table 6: (continued)

Panel B: Acharya and Pedersen liquidity measure

A&P LIQ innovation	0.070	0.060	0.077	0.087	0.087	0.088	0.094
	(3.665)	(2.865)	(3.914)	(4.616)	(4.054)	(4.134)	(4.234)
Rm-Rf	0.089	0.094	0.097	0.092	0.095	0.090	0.094
	(3.568)	(3.829)	(3.894)	(3.684)	(3.776)	(3.383)	(3.635)
Size	0.003	0.002	0.002	0.003	0.003	-0.001	0.000
	(0.310)	(0.112)	(0.123)	(0.281)	(0.320)	(-0.064)	(0.017)
Firm age	0.026	0.029	0.027	0.033	0.030	0.035	0.028
	(1.491)	(1.915)	(1.521)	(2.051)	(1.745)	(2.196)	(1.688)
Growth investment	-0.169	-0.179	-0.158	-0.170	-0.174	-0.175	-0.187
	(-4.197)	(-4.448)	(-3.862)	(-4.517)	(-4.692)	(-4.656)	(-4.639)
A&P LIQ innovation * Size	0.059			0.048	0.050	0.051	0.050
	(4.075)			(3.571)	(2.885)	(3.020)	(2.604)
A&P LIQ innovation * Firm age		0.037		0.033	0.029	0.035	0.034
		(2.196)		(1.971)	(1.709)	(2.130)	(2.078)
A&P LIQ innovation * Growth investment			-0.091	-0.075	-0.082	-0.069	-0.070
			(-1.991)	(-1.759)	(-1.912)	(-1.667)	(-1.712)
Country fixed effects	no	no	no	no	yes	no	yes
Industry fixed effects	no	no	no	no	no	yes	yes
Adj. R2	0.065	0.061	0.062	0.070	0.079	0.080	0.092
N	4403	4403	4403	4403	4275	4118	4037

Table 6: (continued)

Panel C: Sadka liquidity measure							
Sadka LIQ innovation	0.035	0.038	0.026	0.022	0.020	0.020	0.020
	(1.774)	(1.890)	(1.079)	(0.890)	(0.855)	(0.840)	(0.849)
Rm-Rf	0.097	0.098	0.097	0.096	0.096	0.093	0.096
	(3.488)	(3.517)	(3.466)	(3.485)	(3.472)	(3.110)	(3.323)
Size	-0.030	-0.020	-0.019	-0.032	-0.032	-0.038	-0.035
	(-2.403)	(-1.344)	(-1.260)	(-2.602)	(-2.181)	(-3.115)	(-2.317)
Firm age	0.023	0.022	0.024	0.023	0.020	0.027	0.021
	(1.248)	(1.231)	(1.289)	(1.265)	(1.107)	(1.494)	(1.120)
Growth investment	-0.175	-0.167	-0.166	-0.170	-0.177	-0.173	-0.184
	(-3.821)	(-3.698)	(-3.678)	(-3.672)	(-3.761)	(-3.699)	(-3.979)
Sadka LIQ innovation * Size	0.029			0.032	0.033	0.033	0.032
	(4.897)			(5.821)	(4.857)	(5.494)	(4.470)
Sadka LIQ innovation * Firm age		0.017		0.008	0.008	0.007	0.010
		(1.499)		(0.700)	(0.742)	(0.573)	(0.912)
Sadka LIQ innovation * Growth investment			0.045	0.063	0.066	0.079	0.073
			(1.300)	(1.836)	(1.990)	(2.260)	(2.058)
Country fixed effects	no	no	no	no	yes	no	yes
Industry fixed effects	no	no	no	no	no	yes	yes
Adj. R2	0.059	0.056	0.056	0.061	0.071	0.073	0.082
N	4020	4020	4020	4020	3905	3748	3676

Table 7: Liquidity and macro conditions. The table reports the results of OLS regressions. The dependent variable is the Modified IRR (S&P 500 index as reinvestment rate) of individual investments. Explanatory variables include a measure of aggregate liquidity innovations during the investment life, the average excess market return (CRSP value-weighted index minus the risk free rate) during the investment life, a dummy variable that takes the value one if the investment is classified as growth (zero otherwise), and macro-condition measures. Each explanatory variable is expressed as a z-score (subtract mean and divide by standard deviation for each variable). Country and industry fixed effects may be added to the set of explanatory variables. Standard errors are clustered at the investment year level. T-statistics are reported below the coefficients in parentheses. The liquidity variable is either that of Pastor and Stambaugh (Panel A), Acharya and Pedersen (Panel B) or Sadka (Panel C).

Panel A: Pastor and Stambaugh liquidity measure

P&S LIQ innovation	0.096	0.095	0.099	0.093	0.117		0.112	0.117
	(3.759)	(3.740)	(4.009)	(3.533)	(4.776)		(5.580)	(5.890)
Rm-Rf	0.067	-0.002	0.074	0.066			-0.030	-0.040
	(1.986)	(-0.060)	(2.089)	(2.199)			(-0.690)	(-0.871)
Growth investment	-0.066	-0.058	-0.066	-0.056	-0.068	-0.062	-0.054	-0.060
	(-3.692)	(-3.458)	(-3.776)	(-3.289)	(-3.857)	(-3.518)	(-3.063)	(-3.417)
Default spread	0.009						0.047	0.046
	(0.355)						(1.398)	(1.276)
Industrial Production Growth		0.095					0.177	0.186
		(3.151)					(4.292)	(4.607)
(innovation in) IPO volume			-0.019		0.019	0.073	-0.046	-0.044
			(-0.711)		(0.774)	(2.438)	(-1.595)	(-1.460)
Delta VIX				0.003			-0.038	-0.041
				(0.111)			(-1.276)	(-1.284)
Country fixed effects	no	no	no	no	no	no	no	yes
Industry fixed effects	no	no	no	no	no	no	no	yes
Adj. R2	0.068	0.080	0.068	0.065	0.059	0.029	0.092	0.114
N	4403	4403	4402	4286	4402	4402	4286	3937

Table 7: (continued)

Panel B: Acharya and Pedersen liquidity measure

A&P LIQ innovation	0.065	0.072	0.057	0.060	0.072		0.074	0.081
	(2.680)	(3.273)	(2.619)	(2.487)	(2.620)		(3.214)	(3.223)
Rm-Rf	0.084	0.013	0.090	0.100			-0.007	-0.014
	(2.698)	(0.426)	(2.889)	(3.708)			(-0.147)	(-0.289)
Growth investment	-0.066	-0.061	-0.067	-0.058	-0.070	-0.062	-0.054	-0.061
	(-3.673)	(-3.502)	(-3.788)	(-3.311)	(-3.897)	(-3.518)	(-3.036)	(-3.351)
Default spread	-0.025						0.002	0.000
	(-0.844)						(0.046)	(0.002)
Industrial Production Growth		0.113					0.163	0.170
		(3.599)					(3.478)	(3.667)
(innovation in) IPO volume			0.009		0.064	0.073	-0.037	-0.033
			(0.316)		(2.523)	(2.438)	(-1.277)	(-1.098)
Delta VIX				0.019			-0.034	-0.036
				(0.781)			(-1.134)	(-1.095)
Country fixed effects	no	no	no	no	no	no	no	yes
Industry fixed effects	no	no	no	no	no	no	no	yes
Adj. R2	0.057	0.074	0.056	0.055	0.043	0.029	0.078	0.100
N	4403	4403	4402	4286	4402	4402	4286	3937

Table 7: (continued)

Panel C: Sadka liquidity measure

Sadka LIQ innovation	0.034	0.040	0.036	0.047	0.053		0.045	0.046
	(1.756)	(2.001)	(1.787)	(2.254)	(2.621)		(2.384)	(2.340)
Rm-Rf	0.102	0.024	0.082	0.100			-0.016	-0.025
	(2.950)	(0.717)	(2.250)	(3.313)			(-0.288)	(-0.394)
Growth investment	-0.064	-0.057	-0.063	-0.056	-0.063	-0.062	-0.052	-0.057
	(-3.139)	(-2.955)	(-3.155)	(-2.957)	(-3.258)	(-3.518)	(-2.662)	(-2.946)
Default spread	0.002						0.042	0.041
	(0.076)						(1.078)	(0.947)
Industrial Production Growth		0.107					0.161	0.170
		(3.094)					(3.470)	(3.678)
(innovation in) IPO volume			0.030		0.083	0.073	0.013	0.016
			(0.922)		(3.128)	(2.438)	(0.445)	(0.500)
Delta VIX				0.026			-0.026	-0.027
				(0.987)			(-0.694)	(-0.655)
Country fixed effects	no	no	no	no	no	no	no	yes
Industry fixed effects	no	no	no	no	no	no	no	yes
Adj. R2	0.053	0.070	0.055	0.053	0.044	0.029	0.077	0.098
N	4020	4020	4020	3933	4020	4402	3933	3600

Table 8: Correlations and distributions of the traded factors. This table shows the correlation matrix and summary statistics for the (time-series of the) four traded risk factors: the illiquid-minus-liquid factor by Pastor and Stambaugh (2003), the excess market return, HML, SMB. The time period is from October 1975 to December 2007. The frequency is monthly. Returns are in percent.

	IML_PS	Rm-Rf	HML	SMB
Correlations:				
IML_PS	1.000			
Rm-Rf	-0.100	1.000		
HML	-0.276	-0.460	1.000	
SMB	0.042	0.236	-0.341	1.000
Mean	0.375	0.630	0.417	0.241
St. Deviation	4.138	4.320	3.009	3.166
5th percentile	-5.767	-6.410	-3.960	-4.180
Median	0.608	0.940	0.370	0.120
95th percentile	5.530	7.010	5.330	4.800

Table 9: Risk models, alphas, and the cost of capital. The table reports the results of OLS estimation of three different factor models for private equity returns (Panel A) and the resulting alphas and cost of capital (Panel B). In Panel A, the dependent variable is the logarithm of one plus the monthly MIRR minus the log of one plus the risk free rate. Each observation corresponds to a portfolio of investments in private equity formed by date of investment start. Portfolios need to contain at least thirty investments. Each observation is weighted by the square root of the investment duration to correct for unequal variance. Explanatory variables include the Fama and French (1993) three factors (excess market return, HML, SMB) and the illiquid minus liquid portfolio (IML_PS) by Pastor and Stambaugh (2003). Each explanatory variable is computed by taking its average value during the investment's life. All variables are at the monthly frequency. All specifications are run between October 1975 and December 2007. The table also reports the estimate of the residual standard deviation (sigma) and the number of observations (N). In Panel B, the cost of capital is computed as the sum of the average risk free rate plus the products of the factor loadings times the average factor realizations. Alphas and betas are computed using Equations 6 and 7 in the text. The reported values (in %) are annualized by multiplying the monthly estimate by 12. In this panel, we also report the annualized average net risk-free rate. The standard errors for the Panel B estimates are computed using the delta method. T-statistics are given in parentheses.

Model:	Market	FF	PS
Panel A: Risk Models			
IML_PS			0.678
			(3.238)
Rm-Rf	1.001	1.526	1.299
	(6.083)	(4.798)	(4.166)
HML		0.596	0.93
		(1.776)	(2.761)
SMB		0.071	0.034
		(0.252)	(0.127)
Constant	0.005	-0.001	-0.002
	(4.784)	(-0.262)	(-0.626)
Sigma	0.046	0.045	0.043
Adj. R2	0.863	0.866	0.878
N	103	103	103

Model:	Market	FF	PS
Panel B: Alpha and Cost of Capital			
Alpha	8.545%	2.291%	0.181%
	(8.679)	(0.760)	(0.059)
$\beta_{liq} \times \mu_{liq}$			3.110%
			(3.315)
$\beta_{mkt} \times \mu_{mkt}$	7.705%	11.753%	10.013%
	(6.129)	(4.874)	(4.262)
$\beta_{hml} \times \mu_{hml}$		3.043%	4.750%
		(1.811)	(2.828)
$\beta_{smb} \times \mu_{smb}$		0.210%	0.101%
		(0.257)	(0.131)
Risk free rate	5.816%	5.816%	5.816%
Cost of Capital	13.521%	20.822%	23.790%

Table A-I: Altering the number of investments per portfolio: risk models. The table reports the results of OLS regressions factor models for private equity returns. The dependent variable is the logarithm of a measure of performance in excess of the log risk free rate. The performance measure is the modified internal rate of return (MIRR) with S&P 500 reinvestment assumption. Each observation corresponds to a portfolio of I investments in private equity sorted by date of investment start. In Panel A, I is 5. In Panel B, I is 20. In Panel C, I is 50. Each observation is weighted by the square root of the investment duration to correct for unequal variance. Explanatory variables include the Fama and French (1993) three factors (excess market return, HML, SMB) and the illiquid minus liquid portfolio (IML_PS) by Pastor and Stambaugh (2003). Each explanatory variable is computed by taking its average value during the investment's life. All variables are at the monthly frequency. All specifications are run between October 1975 and December 2007. T-statistics are given in parentheses. The table also reports the estimate of the residual standard deviation (sigma) and the number of observations (N).

	Panel A: I = 5			Panel B: I = 20			Panel C: I = 50		
IML_PS			0.651			0.638			0.671
			(3.591)			(3.539)			(2.869)
Rm-Rf	1.007	1.596	1.538	0.948	1.395	1.294	0.989	1.565	1.413
	(5.627)	(6.250)	(6.164)	(6.688)	(5.443)	(5.227)	(5.156)	(3.976)	(3.751)
HML		1.429	1.584		0.719	1.020		0.725	1.028
		(4.257)	(4.795)		(2.450)	(3.466)		(1.823)	(2.626)
SMB		-0.454	-0.162		-0.124	-0.040		-0.030	0.027
		(-1.614)	(-0.567)		(-0.497)	(-0.167)		(-0.085)	(0.082)
Constant	0.006	-0.003	-0.005	0.006	0.000	-0.002	0.005	-0.001	-0.003
	(4.557)	(-1.229)	(-2.094)	(6.003)	(0.035)	(-0.712)	(4.177)	(-0.376)	(-0.814)
Sigma	0.078	0.076	0.074	0.049	0.048	0.046	0.042	0.041	0.039
Adj. R2	0.703	0.723	0.736	0.849	0.853	0.865	0.885	0.888	0.899
N	237	237	237	139	139	139	68	68	68

Table A-II: Altering the number of investments per portfolio: alphas, cost of capital, and expected return. The table reports the cost of capital is computed for each of the models estimated in Table A-I and for the portfolios formed using different values for minimum number of investments (I). The cost of capital is computed as the sum of the average risk free rate plus the products of the factor loadings times the average factor realizations. Alphas and betas are computed using Equations 6 and 7 in the text. The reported values (in %) are annualized by multiplying the monthly estimate by 12. In this panel, we also report the annualized average net risk-free rate. The standard errors for the Panel B estimates are computed using the delta method. T-statistics are given in parentheses.

	Panel A: I = 5			Panel B: I = 20			Panel C: I = 50		
	Market	FF	PS	Market	FF	PS	Market	FF	PS
Alpha	11.898%	1.837%	-1.518%	9.303%	3.128%	0.413%	8.392%	1.348%	-1.154%
	(9.511)	(0.730)	(-0.573)	(10.324)	(1.227)	(0.158)	(7.421)	(0.377)	(-0.319)
$\beta\_liq \times \mu_{liq}$			2.992%			2.928%			3.076%
			(3.625)			(3.599)			(2.977)
$\beta\_mkt \times \mu_{mkt}$	7.773%	12.321%	11.885%	7.300%	10.745%	9.980%	7.612%	12.051%	10.895%
	(5.642)	(6.282)	(6.212)	(6.723)	(5.505)	(5.312)	(5.222)	(4.081)	(3.887)
$\beta\_hml \times \mu_{hml}$		7.307%	8.103%		3.668%	5.207%		3.697%	5.247%
		(4.288)	(4.841)		(2.485)	(3.527)		(1.878)	(2.726)
$\beta\_smb \times \mu_{smb}$		-1.339%	-0.479%		-0.365%	-0.118%		-0.088%	0.080%
		(-1.628)	(-0.574)		(-0.504)	(-0.170)		(-0.088)	(0.085)
Cost of Capital	13.589%	24.105%	28.317%	13.116%	19.864%	23.813%	13.428%	21.477%	25.114%
Risk free rate	5.816%	5.816%	5.816%	5.816%	5.816%	5.816%	5.816%	5.816%	5.816%

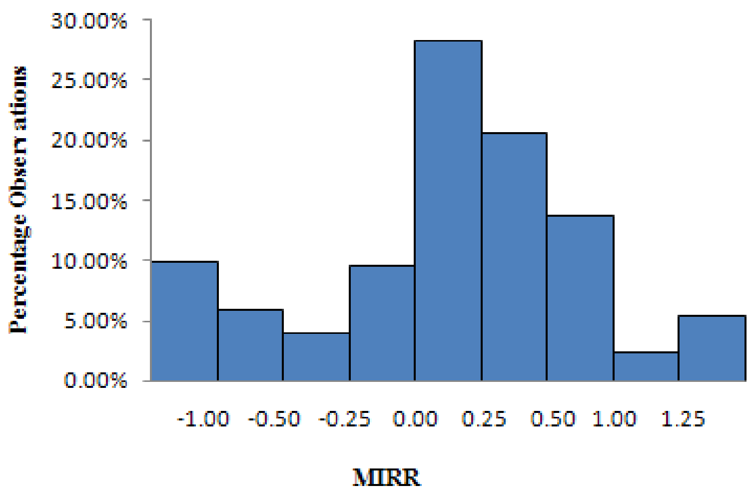

Figure 1: Histogram of MIRR. The figure plots the histogram for the annual MIRR with S&P 500 reinvestment assumption. The first bin contains all the investments with MIRR of -100% annualy. The last bin contain all the investments with MIRR over 125% annually.

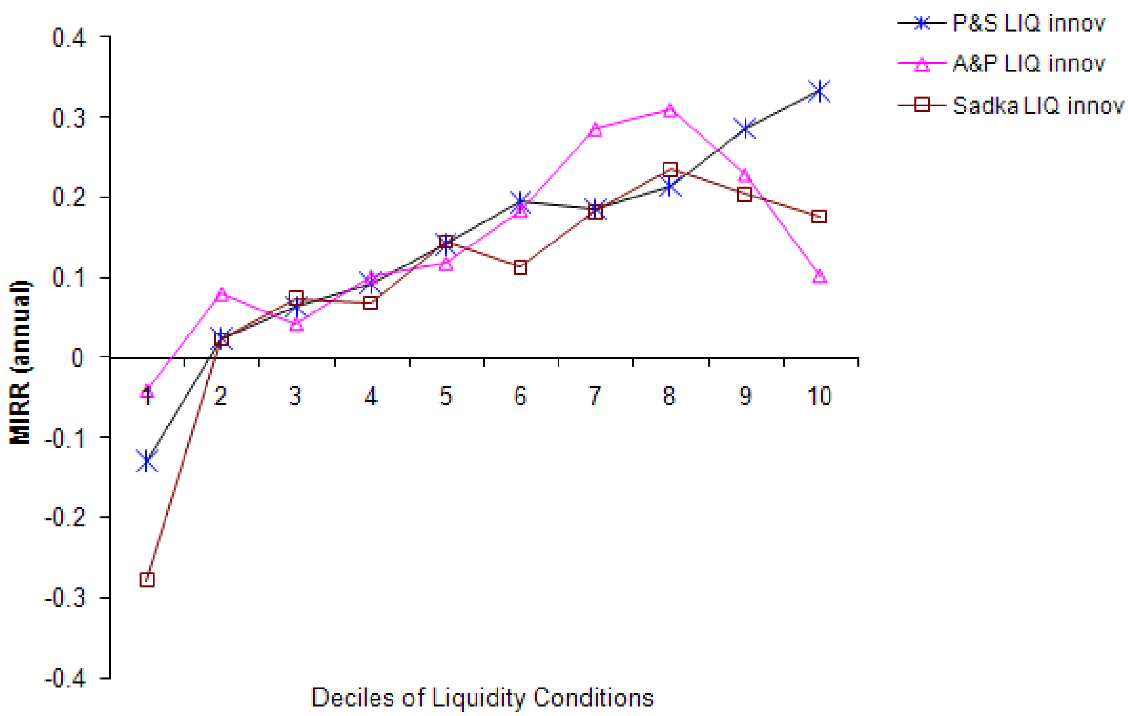

Figure 2: Annual performance by deciles of liquidity innovations. The figure plots the average investment MIRR in each decile of the relevant liquidity innovation. Three measures of liquidity innovations are considered: Pastor and Stambaugh (2003), Acharya and Pedersen (2005), and Sadka (2006).